

Ghost Condensation and Gravity in Higgs Phase

Shinji Mukohyama¹

Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan

Abstract

A tachyon is considered to be sick in the context of particle mechanics, but in field theory just indicates instability of a background. We consider a similar possibility that a ghost in field theory might be just an indication of instability of a background and that it can condense to form a different background around which there is no ghost. We construct a low energy effective field theory based on the derivative expansion around the stable background. Possible applications are discussed, including dark energy, dark matter, inflation and black hole.

1 Introduction

Gravity at long distances shows us many interesting and mysterious phenomena: flattening galaxy rotation curves, dimming supernovae, and so on. These phenomena have been a strong motivation for the paradigm of dark matter and dark energy, i.e. unknown components of the universe which show up only gravitationally. As we essentially do not know what the dark matter and the dark energy are, however, it seems a healthy attitude to consider the possibility that gravity at long distances might be different from what we think we know.

This kind of consideration has been a motivation for attempts for IR modification of gravity, e.g. massive gravity [1] and DGP brane model [2]. However, they are known to have a macroscopic UV scale at around 1000km, where effective field theories break down [3, 4]. This does not necessarily mean that these theories cannot describe the real world, but implies that we need non-trivial assumptions about the unknown UV completion. The recent proposal of ghost condensation [5] evades at least this problem and can be thought to be a step towards a consistent theory of IR modification of general relativity.

In general, if we have scalar fields then there are many things we can play with them. In cosmology, inflation can be driven by the potential part of a scalar field. It is also possible to drive inflation by the kinetic part of a scalar field [6]. On the other hand, scalar fields play important roles also in particle physics. A scalar field is used for spontaneous symmetry breaking and to change force laws in the Higgs mechanism. This is usually achieved by using a potential whose global minimum is charged under the gauge symmetry. The basic idea of ghost condensation is to break a symmetry and change a force law by the kinetic part of a scalar field. In this sense the ghost condensation can be considered as an analogue of Higgs mechanism.

2 Tachyons and Ghosts

A tachyon in particle mechanics is defined as a particle whose speed exceeds the speed of light. It violates causality and, thus, a theory with tachyons is considered to be sick in the context of particle mechanics. However, this is not necessarily true in field theory. In field theory a tachyon is an excitation around a top of a potential. In this case a tachyon just indicates an instability of the background around which the theory is expanded. If the potential has minima at other field values then the theory can be expanded around any one of them and the low energy effective theory is healthy even in the context of particle mechanics. One important point is that we cannot talk about evolution from the unstable background with tachyons to the stable background without tachyons within the context of particle mechanics. In order to describe such an evolution, we need a framework more general than particle mechanics, namely field theory.

¹E-mail: mukohyama@phys.s.u-tokyo.ac.jp

A ghost in field theory can be defined as a field with a wrong sign kinetic term. This is equivalent to say that a ghost is a field with a negative norm. Because of this, a field theory with ghosts is thought to be sick. Indeed, aside from the negative norm, the existence of a ghost indicates instability since the energy associated with the ghost excitation is not bounded from below at least perturbatively in the context of field theory. This situation for ghosts in field theory is somehow similar to that for tachyons in particle mechanics. Hence, it seems natural to expect that a more general, perhaps not-yet-known framework should be able to describe a ghost as just an indication of instability of the background around which the theory is expanded. If this is the case, the general framework should be able to describe the dynamics from a background with ghosts to another background without ghosts. What is important here is that, even without such a framework, we can construct a low energy effective field theory (EFT) around the latter background, which we call ghost condensate. For this reason, we do assume the existence of the more general framework, or a UV completion, but do not need to assume any properties of the UV completion to describe low energy excitations of ghost condensate.

3 Ghost Condensation

The ghost condensation can be pedagogically explained by comparison with the usual Higgs mechanism as in the table shown below. First, the order parameter for ghost condensation is the vacuum expectation value (vev) of the derivative $\partial_\mu \phi$ of a scalar field ϕ , while the order parameter for Higgs mechanism is the vev of a scalar field Φ itself. Second, both have instabilities in their symmetric phases: a tachyonic instability around $\Phi = 0$ for Higgs mechanism and a ghost instability around $\partial_\mu \phi = 0$ for ghost condensation. In both cases, because of the instabilities, the system should deviate from the symmetric phase and the order parameter should obtain a non-vanishing vev. Third, there are stable point where small fluctuations do not contain tachyons nor ghosts. For Higgs mechanism, such a point is characterized by the vev of the order parameter satisfying $V' = 0$ and $V'' > 0$. On the other hand, for ghost condensation a stable point is characterized by $P' = 0$ and $P'' > 0$. Fourth, while the usual Higgs mechanism breaks usual gauge symmetry and changes gauge force law, the ghost condensation spontaneously breaks a part of Lorentz symmetry (the time translation symmetry) and changes linearized gravity force law even in Minkowski background. Finally, generated corrections to the standard Gauss-law potential is Yukawa-type for Higgs mechanism but oscillating for ghost condensation.

	<i>Higgs Mechanism</i>	<i>Ghost Condensation</i>
<i>Order Parameter</i>	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$
<i>Instability</i>	Tachyon $-m^2 \Phi^2$	Ghost $-\dot{\phi}^2$
<i>Condensate</i>	$V' = 0, V'' > 0$	$P' = 0, P'' > 0$
<i>Spontaneous breaking</i>	Gauge symmetry	Lorentz symmetry (Time translation)
<i>Modifying</i>	Gauge force	Gravitational force (in flat background)
<i>New potential</i>	Yukawa-type	Oscillating

For simplicity let us consider a Lagrangian $L_\phi = P(-(\partial\phi)^2)$ in the expanding FRW background with P of the form shown in the upper right part of the table. We assume the shift symmetry, the symmetry under the constant shift $\phi \rightarrow \phi + c$ of the scalar field. This symmetry prevents potential terms of ϕ from being generated. The equation of motion for ϕ is simply $\partial_t[a^3 P' \dot{\phi}] = 0$, where a is the scale factor of the

universe. This means that $a^3 P' \dot{\phi}$ is constant and that

$$P' \dot{\phi} \propto a^{-3} \rightarrow 0 \quad (a \rightarrow \infty) \quad (1)$$

as the universe expands. We have two choices: $P' = 0$ or $\dot{\phi} = 0$, namely one of the two bottoms of the function P or the top of the hill between them. Obviously, we cannot take the latter choice since it is a ghostly background and anyway unstable. Thus, we are automatically driven to $P' = 0$ by the expansion of the universe. In this sense the background with $P' = 0$ is an attractor. Thus, if there were inflation(s) (irrespective of whether it is the usual potentially driven inflation, k-inflation or ghost inflation) in the early universe then P' is set to an extremely small value.

Now let us consider a small fluctuation around the background with $P' = 0$. For $\phi = M^2 t + \pi$, the quadratic action for π coming from the Lagrangian P is $\int d^4x [P'(M^4) + M^4 P''(M^4)] \dot{\pi}^2 - P'(M^4) (\nabla \pi)^2$. By setting $P'(M^4) = 0$ we obtain the time kinetic term $M^4 P''(M^4) \dot{\pi}^2$ with the correct sign. Unless the function P is fine-tuned, P'' is non-zero at $P' = 0$. This means that the coefficient of the time kinetic term is non-vanishing and, thus, we do not have the strong coupling issue which the massive gravity and the DGP brane model are facing with. On the other hand, the coefficient of $(\nabla \pi)^2$ vanishes at $P' = 0$ and the simple Lagrangian P does not give us a spatial kinetic term for π . However, this does not mean that there is no spatial kinetic term in the low energy EFT for π . This just says that the leading spatial kinetic term is not contained in P and that we should look for the leading term in different parts. Indeed, other terms like $\tilde{P}((\partial\phi)^2)Q(\square\phi)$ do contain spatial kinetic terms for π but the spatial-derivative expansion starts with the fourth derivative: $(\nabla^2 \pi)^2 + \dots$. If there is a non-vanishing second-order spatial kinetic term $(\nabla \pi)^2$ then it can be included in P by redefinition and the redefined P' goes to zero by the expansion of the universe as shown above. Namely, the expansion of the universe ensures that the spatial-derivative expansion starts from $(\nabla^2 \pi)^2 + \dots$. Combining this spatial kinetic term with the previously obtained time kinetic term and properly normalizing π , we obtain the low energy effective action of the form

$$\int d^4x \left[\frac{1}{2} \dot{\pi}^2 - \frac{\alpha}{M^2} (\nabla^2 \pi)^2 + \dots \right], \quad (2)$$

where α is a dimensionless parameter of order unity. One might worry that other (nonlinear) terms in effective theory such as $\dot{\pi}(\nabla \pi)^2$ might mess up the effective action. In fact, it turns out that all such terms are irrelevant at low energy [5]. An important fact to show this is that the scaling dimension of π is not the same as its mass dimension 1 but is 1/4, reflecting the situation that the Lorentz symmetry is broken spontaneously. Moreover, it is also straightforward to show that all spurious modes associates with higher time derivative terms such as $(\ddot{\phi})^2$ have frequency above the cutoff M and, thus, should be ignored. In this sense, we are assuming the existence of a UV completion but not assuming any properties of it. Finally, it must be noted that the effective action of the form (2) is stable against radiative corrections. Indeed, the only would-be more relevant term in the effective theory is the usual spatial kinetic term $(\nabla \pi)^2$, but its coefficient P' is driven to an extremely small value by the expansion of the universe even if it is radiatively generated.

The effective action (2) would imply the low energy dispersion relation for π is $\omega^2 \simeq \alpha k^4/M^2$. However, since the background spontaneously breaks Lorentz invariance, π couples to gravity in the linearized level even in Minkowski or de Sitter background. Hence, mixing with gravity introduces an order M^2/M_{pl}^2 correction to the dispersion relation. As a result the dispersion relation in the presence of gravity is $\omega^2 \simeq \alpha k^4/M^2 - \alpha M^2 k^2/2M_{pl}^2$. This dispersion relation leads to IR modification of gravity due to Jean's instability. Note that there is no ghost around the stable background $P' = 0$ and the Jeans's instability is nothing to do with a ghost.

As stated in the end of Sec. 2, we actually do not need to specify a concrete way of the spontaneous symmetry breaking in order to construct the EFT around the stable background. In this sense, the ghost around $\phi = 0$ has nothing to do with the construction of the EFT around $P' = 0$. Indeed, it is suffice to assume the symmetry breaking pattern, namely from the full 4D Lorentz symmetry to the 3D spatial diffeomorphism [5].

Note that the ghost condensate provides the second most symmetric class of backgrounds for the system of field theory plus gravity. The most symmetric class is of course maximally symmetric solutions: Minkowski, de Sitter and anti-de Sitter. The ghost condensate minimally breaks the maximal symmetry and introduces only one Nambu-Goldstone boson.

4 Possible Applications

Dark energy: In the usual Higgs mechanism, the cosmological constant (cc) would be negative in the broken phase if it is zero in the symmetric phase. Therefore, it seems difficult to imagine how the Higgs mechanism provides a source of dark energy. On the other hand, the situation is opposite with the ghost condensation: the cc would be positive in the broken phase if it is zero in the symmetric phase. Hence, while this by itself does not solve the cc problem, this can be a source of dark energy.

Dark matter: If we consider a small, positive deviation of P' from zero then the homogeneous part of the energy density is proportional to a^{-3} and behaves like dark matter. Inhomogeneous linear perturbations around the homogeneous deviation also behaves like dark matter. However, at this moment it is not clear whether we can replace dark matter with ghost condensate. We need to see if it clumps properly. Ref. [8] can be thought to be a step towards this direction.

Inflation: We can also consider inflation within the regime of the validity of the EFT with ghost condensation. In the very early universe where H is higher than the cutoff M , we do not have a good EFT describing the sector of ghost condensation. However, the contribution of this sector to the total energy density ρ_{tot} is naturally expected to be negligible: $\rho_{ghost} \sim M^4 \ll M_p^2 H^2 \simeq \rho_{tot}$. As the Hubble expansion rate decreases, the sector of ghost condensation enters the regime of validity of the EFT and the Hubble friction drives P' to zero. If we take into account quantum fluctuations then P' is not quite zero but is $\sim (H/M)^{5/2} \sim (\delta\rho/\rho)^2 \sim 10^{-10}$ in the end of ghost inflation. In this way, we have a consistent story, starting from the outside the regime of validity of the EFT and dynamically entering the regime of validity. All predictions of the ghost inflation are derived within the validity of the EFT, including the relatively low- H de Sitter phase, the scale invariant spectrum and the large non-Gaussianity [7].

Black hole: In ref. [9] we consider the question “what happens near a black hole?” A ghost condensate defines a hypersurface-orthogonal congruence of timelike curves, each of which has the tangent vector $u^\mu = -g^{\mu\nu}\partial_\nu\phi$. It is argued that the ghost condensate in this picture approximately corresponds to a congruence of geodesics and the accretion rate of the ghost condensate into a black hole should be negligible for a sufficiently large black hole. This argument is confirmed by a detailed calculation based on the perturbative expansion w.r.t. the higher spatial kinetic term. The essential reason for the smallness of the accretion rate is the same as that for the smallness of the tidal force acted on an extended object freely falling into a large black hole.

Acknowledgements

The author would like to thank Nima Arkani-Hamed, Hsin-Chia Cheng, Paolo Creminell, Markus Luty, Jesse Thaler, Toby Wiseman and Matias Zaldarriaga for collaboration on the subject.

References

- [1] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. **A173**, 211 (1939).
- [2] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. **B485**, 208 (2000).
- [3] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Annals Phys. **305**, 96 (2003).
- [4] M. A. Luty, M. Porrati and R. Rattazzi, JHEP **0309**, 029 (2003).
- [5] N. Arkani-Hamed, H.-C. Cheng, M. A. Luty and S. Mukohyama, JHEP **0405**, 074 (2004).
- [6] C. Armendariz-Picon, T. Damour and V. Mukhanov, Phys. Lett. **B458**, 209 (1999).
- [7] N. Arkani-Hamed, P. Creminelli, S. Mukohyama and M. Zaldarriaga, JCAP **0404**, 001 (2004).
- [8] N. Arkani-Hamed, H.-C. Cheng, M. A. Luty, S. Mukohyama, and T. Wiseman, in preparation.
- [9] S. Mukohyama, hep-th/0502189, to appear in Phys. Rev. **D**.